**Bank networks and systemic risk**

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**Abstract**

This paper presents two bank networks with and without systemic risk. An analytical approach based on Freixas, Parigi and Rochet’s (2000) work is proposed. The theoretical framework uses matrix algebra and network graphs. In this paper, banks are represented by nodes and links are the credit lines. A credit chain is explored to examine how a bank network could collapse by insolvency of one bank. One main setting of this paper is the debt distribution in the bank network maintains the bank system integrity.

**Keywords**: bank networks, credit lines, credit chains, bank solvency, systemic risk.

**JEL Classification: G22, L14**.

**1. Introduction**

It is a well-known fact that most of the economic crises have their origin in the bank system bad functioning. Government bailouts are expected when bank networks signal problems in the financial system. For the sake of simplicity, this paper deals with a simple bank network composed by three banks. Each bank represents the network nodes, this facilitates the different problems modelling that they can face. This simple bank system could be scaled to represent a complex bank network. The objective of this paper is to model in a simple fashion the different interactions that could arise among banks under different risk conditions.

The approach use in this paper comes mainly from game theory, and the modeling is carried out with matrices and network graphs. Also, this approach is inspired on the works of Freixas *et al*. (2000), Freixas and Rochet (2008), Freixas (2010), Freixas and Laux (2011), Freixas and Rochet (2012), Bolton *et al*. (2016), Freixas and Ma (2015a). It is worth mentioning that this type of research started to strongly develop after the global financial crisis of 2007-2009. This research represents an alternative venue to understand the economic phenomena behind financial crises, which have an origin in the banking system.

In a brief literature review on this topic, it was found that Battiston, *et al.* (2012) investigate endogenous systemic risk and financial system architecture to find resilient financial networks, and propose appropriate policy responses. Allen and Gale (2000) study financial system as an equilibrium phenomenon. Here, liquidity preference shocks could cause a financial region contagion, where interregional claims structure could provide insurance against adverse liquidity preference shocks. Boss, *et al*. (2004) provide an empirical analysis of the network structure of the Austrian interbank market and find that the degree distributions of the interbank network follow power laws. Also, they find that a defaulting single bank can affect the network structure and the stability of the banking system. For Cont, *et al*. (2013) the potential for contagion is gauge for the contagion index in a network of interlinked financial institutions. Their empirical exercise comprehends financial institutions in Brazil for the years of 2007 and 2008, emphasizing, the heterogeneity in the network structure and counterparty exposures to explain systemic institutions importance. Ricciardi *et al*. (2022) study complex dynamical interbank network down to a low-dimensional effective system by applying the degree-weighted and spectral reduction methods to find reliable estimates of systemic risk in the market.

In the case of Soramäki, *et al.* (2006) the topology of the interbank payments transfers, between commercial banks over the Fedwire Funds Service display compactness and low connectivity with money-center banks. They found that this network exhibits a scale-free degree distribution, with substantial changes in the aftermath of September 11, 2001. Also, they found that the commercial banks use interbank payments to allocate capital and manage their risk exposures and facilitate clearing and settlement.

Freixas and Holthausen (2004) develops a model with liquidity shocks to find an equilibrium with unsecured integrated markets. In the case of Freixas and Ma (2014) credit risk is modeled based on borrowers’ moral hazard. Also, they examine how banks optimally adjust their leverage considering various risks. Freixas (2018) studies the likelihood of a systemic crisis increases with an excessive credit growth, and the emergence of financial bubbles.

For its part Acemoglu *et al.* (2015a) find that the same factors -dense interconnections and shocks magnitude- that contribute to resilience in financial networks under certain conditions, may function as significant sources of systemic risk under others. In other paper for the same year Acemoglu *et al.* (2015b) study a unified framework to study how networks interactions can function as a mechanism for propagation and amplification of microeconomic shocks. Acemoglu *et al.* (2015c) provide a framework to study the formation of financial networks and investigate the interplay between endogenous financial networks and the banks’ lending incentives, together with the emergence of systemic risk.

A venue of the bank network literature devotes its effort to study the defaulting effects of one bank and/or the major bank participant. For example, González-Avella, *et al.* (2016) find that the interbank market is responsible for efficient liquidity allocation: when one bank is insolvent, it could propagate a financial contagion and introduce the possibility for systemic risk in the market. Angelini, *et al.* (1996) investigates systemic risk in the netting system, where a bank experiences sudden liquidity or solvency problems that may prevent settlement of its direct creditors’ claims, which may in turn jeopardize settlement of other institutions and may have a “domino effect.” Bech and Soramäki (2004) study the impact of a bank failure on other bank system participants. This failure may become contagious and affect the bank system function, and therefore the financial system at large. Devriese and Mitchell (2005) analyze the largest player default and its potential impact on the securities settlement system, which may cause in turn a major market disruption even when ample liquidity is provided: whereas a broad program of securities borrowing, and lending might help during market disruptions, it is when participants will be least willing to lend securities. Humprey (1986) studies how failures to settle one or more large debit daylight overdraft -intraday borrowing by a bank at a zero interest cost in a bank network- disrupt the smooth operation of the payments system and financial markets. For him possible remedials for the disruption are payments finality, as one way to reduce the risk of a settlement failure. Also, he considers these remedials together with a bilateral net credit limit, and sender net debit caps. van-Lelyvel and Liedorp (2006) found using survey data (obtained from ten banks substituted in the interbank-lending matrix), that the bankruptcy of one of the large banks will put a considerable burden on the other banks, but it will not lead to a complete collapse of the interbank market.

This paper is organized as follows: section 2 is devoted to present the methodology, section 3 develops a bank diversified regimen, section 4 develops the bank less resilient regimen. In the last section a conclusion is put forward.

**2. Methodology**

The theoretical framework is a game theory setting where agents under analysis are banks. For this model three banks are considered in diverse locations (. Next, a triangular graphical representation of this network system is presented, where each bank is placed in a vertex for the sake of simplicity.

Figure 1. Spatial bank distribution in a triangular network

Source: own elaboration.

One factor of the bank’s heterogeneity consists in their distinct locations. The network links are the credit lines ( for , ( for , and ( for . The next step is the representation of these credit lines for each bank. In the next figure these credit lines are represented for each bank.

Figure 2. Connections among banks through credit lines

Source: own elaboration.

Suppose that a good is traded among banks, and that it can be considered as cash. This is good-unit type no perishable *à la* Arrow-Debreu clearing market model (Arrow and Debreu (1954), Arrow (1951), Debreu (1951), Debreu (1952)). Three clients or consumers ( are endowed at day zero with ( units of this good. The banks do not have assets to lend by themselves, but the deposits that each consumer made on them. This good-unit is divisible and, as we say before, it can be considered as cash.

There are three periods on which banks can experiments movements in their quantity of good-unit, and in their credit lines. At time zero, each bank receives deposits of one unit of the good from one of the three consumers. At time one, banks can lend their deposits to investors and receive a known interest rate () over them, and/or experiment endogenous withdrawals of size determined by the consumers in different point-locations (. At time two, markets clear, meaning that investors pay their debts at rates pertaining to the bank network under analysis. To keep symmetry, it is assumed that there are three types of investors ( and banks returns units of goods to consumers as indicated in the Arrow-Debreu model when markets clear. The interest is retained by the bank network for the concept of intermediation. This implied that consumers saving is equal to zero.

The dynamic of the game depends on the assumptions made at time zero. The outcomes of the game could be as many as the number of combinations of the initial conditions. However, this paper focuses in two principal outcomes when the game interrelations among the network banks could be label as “diversified” and “less resilient” regimens.

The dynamics of the game theory for this setting could be represented by a matrix at day zero and dimension of 3 by 3, with a structure like the identity matrix . Matrix at day zero represents the endowments of the good-unit for each consumer. Matrix at day zero represents the allocations of the good-unit for each consumer. By symmetry matrix also represent the allocation of the good-unit for each consumer in each bank . Matrix represents the credit lines among banks:

, and

According with the assumptions of this game, the interactions of banks could be treated analytically by means of matrix algebra and produce a systemic risk indicator .

**3. Banks diversified regimen**

The credit lines are created when a consumer withdraws at time 2 his deposit or part of it in a different location. In this scenario banks are known to be solvent, as there is not news that contradicts this belief. This is a complete network graph with the following representation:

Figure 3. Banks diversified regimen

Source: own elaboration.

Note that in figure 3 all credit lines are depicted, along with its proper direction as explained in section 2.

Analytically the liquidity on this network is represented as:

(1)

Equation (1) represents that for any withdrawal of consumer , this is equal in amount to the interest rate that it pays. Equation (1) can be seen also as a ratio:

(2)

where stands for leverage for each bank . When =1, this means that , as in equation (1).

Consider now the following allocation on the unit circle of the good-unit with respect to matrix

(3)

where matrix *E* represents different allocations of the good-unit in different banks at any time. As matrix *E* is only a weighted average of the good-unit allocations, then . To check if the network bank is in balance, we can apply the accountability bank criterion: bank assets equals investor debts, and =1 then:

(4)

Thus, if (4) holds the leverage expressed in equation (2) equals 1. Here, the systemic risk indicator is also equal to one. In this case, we can say that the bank network is solvent: (((. We can call this last expression as a credit line equality, or a bank diversified path. In this path, there is not systemic risk to distribute among banks as all banks are equally solvent. There is not systemic risk contagion on this bank network

**4. Banks less resilient regimen**

Contrary to the case analyzed in the previous section, here the bank network is not solvent. That is to say, the bank network has problems with liquidity.

Following Freixas and Rochet (2000), considered that only one bank, *i.e*., is insolvent and/or have liquidity problems. This is an incomplete network graph with the following representation:

Figure 4. Banks less resilient regimen

Source: own elaboration.

The analytical condition of this network could be placed as:

(5)

The accountability criterion explained in the previous section, but now applied for also delivers the same inequality expressed in equation (5) to represent all the bank network:

(6)

Equation (6) implies that in the bank network there are more debts than assets even only one bank is insolvent. If this news is made public, it could cause consumers loss of trust in the banking system and create a run. If the severity of the run is critical, it could cause a systemic risk contagion, follow by a financial crisis and worst over all these downturns, a real economic crisis. To measure the severity of the insolvency, the systemic risk parameter could be a good approximation. The parameter range is , where represents zero systemic risk, and represents an increasing systemic risk as it approaches infinity. As approaches , the bank system becomes less and less resilient. The systemic risk parameter is defined as the inverse of the leverage ratio expressed in equation (2).

(7)

The above equation expresses any withdrawal of consumer as a percentage of the interest rate that it pays. The credit lines, with insolvent, are represented with the following matrix:

This matrix represents the case when is insolvent. The insolvency of leaves the bank network to operate with only two endowment good-units (those from and ). These two good-units allocation could be represented in the following matrix

All the operations that were carried out in the bank diversified path can be performed in this case, but with more fragility. That is to say, if the bank network increases its good-unit/money velocity it could cover the credit lines of , even they have already disappeared by assumption. In this case the bank system could operate normally if it provides a credit chain to . A credit chain here is defined as a subnetwork that supports the matrix , with the purpose that the bank network works without solvency problems. The news about the insolvency of could become rumors, and the risk of a systemic contagion, and a financial crisis is diminished. The bank one insolvency consequences are avoided as the bank network continues operating as it belong to the banks diversified network regimen. It is important to say, that the systemic risk in this bank network regimen remains the same as before, but under systemic risk contagion control. To see this analytically consider the following next equation (8), with some modification, with primes, to express an equality:

(8)

where equals two good-units available in the bank system, and it can be treated as a scalar in a matrix representation, *i.e*., with . Canceling common terms on equation (8) we have: . To control the liquidity constraint that the insolvency of is imposing in this bank network, consider the following condition:

(9)

where . Also, suppose that provides the prime distribution of over the bank network, . This is depicted in the following equation (10):

= (10)

The sum of , which coincides with the assume value of , and with the number of good-unit in this bank system. Thus, the systemic risk parameter calculation proceeds as follows:

=

Remember than in the banks diversified regimen In this case the parameter value indicates no risk. Now the banks less resilient regimen is so, since the systemic risk parameter is , which indicates a bigger systemic risk.

In what follows, we compute the new credit lines distribution in matrix in order to see how the systemic risk is distributed in the entire bank network. The purpose to do this is to demonstrate that the system could work as it has not solvency problems. Matrix computation is shown bellow

Adding all the individual’s components of This result represents an endowment of tree good-units allocation in the less resilient regimen. This regimen can operate as it has the initial endowment of the previous regimen, because the velocity of money has increased from one in the banks diversified regimen to two in the banks less resilient regimen.

Thanks to the mechanism propose in here the banks less resilient regimen operates under the conditions of a bank diversified regimen, and without systemic risk contagion. This last regimen only uses only two good-units but have credit lines that covers the operations made with three good-units. Is noteworthy that the matrix is multiplied by 2 and for this fact the six initial credit lines are reestablished in the bank less resilient regimen. This result was achieved by using only two good-units pertaining to and , since was insolvent by assumption.

**Conclusion**

In the first regimen, when all banks are solvent, there is not systemic risk contagion as the systemic risk parameter is equal to one. When one bank is insolvent and/or have liquidity problems, the rest of the bank network could cover its credit lines for the insolvent bank no to default and avoiding systemic risk contagion and financial crisis.

It is worth mentioning that the matrix in the second regimen, banks less resilient regimen, is multiplied by 2. This means that the velocity of money/good-units is two. In other words, the matrix has to circulate two times to reestablish the six initial credit lines of the first regimen. The second regimen gives an example on how this could happen under a debt’s distribution control in the bank network. Although, the network bank in the second regimen operates with normality, without systemic risk contagion, its fragility has increased. This is because the measure of the systemic risk in the banks less resilient regimen gives a value of .

The mechanism implemented in this paper allows the bank network to resume control and operate with normality, although one bank is defaulting. This mechanism allows the second regimen to have control over systemic risk contagion.

A healthy network bank is desirable. Regulations are expected to prevent having insolvent banks that could lead to fragile bank networks, where if controls are not implemented, and to financial and real economic collapses as Freixas (2010), Freixas *et al*. (2015a, 2015b), Eslava and Freixas (2021) mention. The systemic risk parameter is a measure that could help control the bank network risk by adjusting the money/good-units/cash velocity. This paper mechanism could adjust a banks less resilient regimen debt distribution by allowing it to work under zero systemic risk conditions. Thus, although there is a defaulting bank the bank network operates under normal conditions, as in the first regimen. If the systemic risk contagion is avoided through this mechanism, then the consumer confidence in the bank system is affirmed and it can avoid panics/runs, which are associated with financial and real economic crises.

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